THEORY OF PLASMA ACCELERATION IN ELECTRIC AND MAGNETIC FIELDS

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The problem of the acceleration of a plasma in crossed electric and magnetic fields under the simplest physical conditions suitable for comparison with experiment is considered. Analytical expressions are obtained for the velocity of the electrons, the value of the resonance acceleration zone, and the increment of the potential of the accelerated plasma.

Suppose a cold plasma is situated in a nonuniform magnetic field

$$\mathbf{B}\{{}^1/_2\delta B_0x,\ {}^1/_2B_0\delta y,\ B_0(1-\delta z)\},\qquad \delta=|dB_0/dz|\,B_0^{-1}$$

where B_0 is the magnetic field strength at the beginning of the resonator with $z=z_0$, δ is the magnetic field nonuniformity factor, and x, y, z are coordinates connected with the laboratory system of coordinates.

A high-frequency field

$$E_x=E_0(y,z)\cos(\omega t+\varphi),\qquad E_y=E_z=0$$

is excited in the resonator, where E_0 is the amplitude, ω is the frequency, and φ is the initial phase of the electric field at the instant $t=0$ when the accelerating particles enter the resonator.

At a frequency ω close to the Larmor frequency ω_c of rotation of an electron in a magnetic field ($\omega_c = eB_z/mc$, where e and m are the charge and mass of the electron, respectively, B_z is the component of the magnetic field along the z axis, and c is the velocity of light), resonance acceleration of the electron occurs in the resonator, described by the equations

$$\frac{dv_x}{dt}=-\frac{e}{m}E_0\cos(\omega t+\varphi)-\frac{e}{mc}v_yB_z,\qquad \frac{dv_y}{dt}=\frac{e}{mc}v_xB_y \tag{1}$$

where v_x and v_y are the components of the electron velocity along the x and y coordinates, and B_y is the component of the magnetic field along the y axis. By changing to the complex variable $r=x+iy$, where i is the square root of -1, Eq. (1) becomes

$$dv_r/dt-i\omega_cv_r=-\gamma\cos(\omega t+\varphi) \tag{2}$$

where $v_r=v_x+iv_y$, $\gamma=eE_0/m$.

Its solution

$$v_r=\frac{\gamma_i}{2}\left[\frac{\exp i(\omega t+\varphi)-\exp i(\omega_ct+\varphi)}{\omega_c-\omega}+\frac{\exp[-i(\omega t+\varphi)]-\exp i(\omega_ct-\varphi)}{\omega_c+\omega}\right] \tag{3}$$

enables us to determine the velocity and energy of an accelerated electron. The ions, because of their inertia, do not experience resonance acceleration in the high-frequency electric field.

Under the action of the magnetic field gradient, the accelerated electrons begin to be drawn into the region of weaker magnetic fields along the z axis. As the electrons move they attract ions with which they combine as a result of the electric attractive forces. This effect becomes particularly large in a narrow

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region d close to resonance $z = z_p$, as the Larmor frequency ω_c approaches the frequency of the electric field ω, which leads to resonance acceleration of the plasma.

The motion of the plasma along the z axis can be considered using the usual equations for its components, namely electrons and ions

$$\frac{dv_{ze}}{dt} = \frac{\partial v_{ze}}{dt} + (v_e\nabla)\, v_{ze} = -\frac{e}{m} E_\rho + \frac{e}{mc}(v_{xe}B_y - v_{ye}B_x) \tag{4}$$

$$\frac{dv_{zi}}{dt} = \frac{\partial v_{zi}}{\partial t} + (v_i\nabla)\, v_{zi} = \frac{e}{M} E_\rho \tag{5}$$

where E_ρ is the field strength of the space charge which occurs as the accelerated electrons travel along the z axis, v_{ze} and v_{zi} are the velocity of the electrons and ions in the same direction, and M is the mass of an ion.

Taking into account the dependence of B_x and B_y on the coordinates, we can transform the expression

$$\frac{e}{mc}(v_{xe}B_y - v_{ye}B_x) = \frac{\delta\omega_0}{2}\langle \dot{x}y - \dot{y}x\rangle = \frac{\delta\omega_0}{2}\langle \mathrm{Re}(-i\dot{r}r^*)\rangle$$

by introducing the complex quantities r, $dr/dt = \dot{r}$, and r^*, which is conjugate to r,

$$\frac{dv_{ze}}{dt} = -\frac{e}{m} E_\rho + \frac{\delta\omega_0}{2}\langle \mathrm{Re}\, r\, \mathrm{Im}\, v_r - \mathrm{Im}\, r\, \mathrm{Re}\, v_r\rangle \tag{6}$$

where $\omega_0 = eB_0/mc$, and r and v_r are found from Eq. (3).

Substituting the expression for E_ρ from Eq. (5) and assuming, in the case of a quasineutral plasma,

$$n_i = n_e = n_0, \qquad n_i v_{zi} = n_e v_{ze}, \qquad v_{zi} = v_{ze} = v_z$$

where n_e and n_i are the electron and ion densities of the plasma, respectively, we have

$$\frac{dv_z}{dt} = \frac{m}{m+M}\,\frac{\delta\gamma^2}{2}\,\frac{3\omega_c^2 + \omega^2 + (\omega_c^2 - \omega^2)\langle\cos 2\varphi\rangle}{4(1-\delta z)(\omega_c^2 - \omega^2)^2} \tag{7}$$

where $\langle\cos 2\varphi\rangle$ is the average statistical value, connected with the phase φ at the instant the electron enters the resonator. If, for example, we set it equal to unity (all the phases are equiprobable), we have

$$\frac{dv_z}{dt} = \frac{m}{M}\,\frac{\delta\gamma^2}{2}\,\frac{\omega_0\omega_c}{(\omega_c^2 - \omega^2)^2} \tag{8}$$

For the steady-state case $\partial v_z/\partial t = 0$, we obtain

$$v_z^2 = \frac{m}{M}\,\frac{\delta\gamma^2}{4}\,\omega_0^2 \int_0^z \frac{(1-\delta z)\,dz}{[\omega_0^2(1-\delta z)^2 - \omega^2]^2} \tag{9}$$

whence we find the increase in plasma energies along the path from $z = 0$ to any $z < z_p$, where z_p is the coordinate of the exact equality of ω and ω_c.

The space-charge field E_ρ for any $z < z_p$ can be found from expressions (5) and (8) using the equation $v_{zi} = v_z$

$$E_\rho = \frac{M}{e}\,\frac{dv_z}{dt} = \frac{m}{e}\,\frac{\delta\gamma^2}{2}\,\frac{\omega_0^2(1-\delta z)}{[\omega_0^2(1-\delta z)^2 - \omega^2]^2} \tag{10}$$

The space-charge field is related to the space-charge distribution by Poisson's equation, and while the plasma is being accelerated, the change in the field E_ρ along the x and y axes is much less than its change along the z axis

$$\frac{dE_\rho}{dz} = \frac{m}{e}\,\frac{\delta^2\gamma^2\omega_0^2}{2}\,\frac{3\omega_0^2(1-\delta z)^2 + \omega^2}{[\omega_0^2(1-\delta z)^2 - \omega^2]^3} = 4\pi e\,(n_i - n_e) \tag{11}$$

In the region $z < z_p$ we can assume that the plasma remains essentially in a quasineutral state. However, as $z \to z_p$, $\omega_c \to \omega$ and, as can be seen from expressions (7)-(11), both the plasma velocity v_r and the space-charge field E_ρ tend to infinity together with all the derivatives. The difference between the ion and electron densities (11) also become infinite. But, in a practical plasma, this difference cannot exceed the absolute value of the charge density n_0. For such a difference in density one cannot assume the plasma to be quasineutral, which means that expressions (7)-(11) cannot be applied to this region, in which the space-charge field E_ρ is in fact replaced by a tensor mass (m + M), as in the papers by Canobbio [1-3]. This is inadmissible for the resonance region in which the quasineutral state of the plasma is disturbed.

Thus, in the resonance region d only the following set of equations can used:

$$\frac{dv_{ze}}{dt} = -\frac{e}{m} E_\rho + \frac{e}{mc}(v_x B_y - v_y B_x) \tag{12}$$
$$\frac{dv_{zi}}{dt} = \frac{e}{M} E_\rho, \qquad \frac{dE_\rho}{dz} = 4\pi e (n_i - n_e)$$

In system (12) the shape of the space-charge distribution curve $(n_i - n_e)$ in the resonance region is unknown. It is very difficult to obtain it experimentally, because of the small value of d and the high electron velocities.

However, it can be approximated in fairly general form, for example, as follows:

$$(n_i - n_e)/n_0 = A\xi \exp(1 - \alpha\xi^2)$$

where $\xi = (z_p - z)/(z_p - z_H)$, and it varies from 0 to 1 for $z_H \le z \le z_p$, where z_H is the beginning of the resonance region, i.e., the region in which considerable disturbance of the quasineutral state of the plasma occurs, in which the difference between the electron and ion densities becomes comparable in value with the density itself, and A and α are coefficients. The results of the solution of Eq. (12), assuming the above approximation, can be compared with experiment.

For the maximum difference between the densities $(n_i - n_e)_{max} = n_0$, which corresponds to the condition

$$\xi_{max} = 1/\sqrt{2\alpha} \quad \text{or} \quad z_{max} = [z_p - (z_p - z_H)]/\sqrt{2\alpha}$$

we obtain

$$A\xi \exp(1 - \alpha\xi^2)_{max} = 1$$

whence

$$A = \sqrt{2\alpha} \exp(-{}^1/_2)$$

When solving the system of equations for the ions

$$\frac{dv_{zi}}{dt} = \frac{e}{M} E_\rho, \qquad \frac{\partial v_{zi}}{\partial t} = 0 \tag{13}$$
$$dE_\rho/dz = 4\pi e n_0 A\xi \exp(1 - \alpha\xi^2)$$

for E_ρ, with the above assumptions, we obtain an integral of the type

$$E_\rho = K\int \exp(-q)\, dq + C,$$

where C = 0, since for large values of ξ the space-charge field tends to zero. Thus

$$E_\rho = 4\pi e n_0 d \exp({}^1/_2 - \alpha\xi^2)/\sqrt{2\alpha} \tag{14}$$

where $d = z_p - z_H$ is the value of the resonance zone.

The coefficient α and the value of the resonance zone d can be obtained from the conditions of the boundary of the region d with $z = z_H$, since the value of E_ρ and its derivative dE_ρ/dt defined by Eqs. (10), (11), and (14) on the boundary of the zone must be identical. Consequently, for $z = z_H$, $\xi = 1$,

$$E_\rho = \frac{m\delta\gamma^2\omega_0^2 (1 - \delta z_H)}{2[\omega_0^2 (1 - \delta z_H)^2 - \omega^2]^2} = 4\pi e n_0 d \exp({}^1/_2 - \alpha)/\sqrt{2\alpha} \tag{15}$$

$$\frac{dE_\rho}{dz} = \frac{m\delta^2\gamma^2\omega_0^2}{2e} \frac{[3\omega_0^2 (1 - \delta z_H)^2 + \omega^2]}{[\omega_0^2 (1 - \delta z_H)^2 - \omega^2]^3} = 4\pi e n_0 \sqrt{2\alpha} \exp({}^1/_2 - \alpha) \tag{16}$$

Hence, taking into account the fact that

$$(1 - \delta z_H) = \omega/\omega_0 + \delta d, \qquad \delta d < \omega/\omega_0$$

we obtain the value of the resonance zone, and $\Delta\Psi$ is the increment in the plasma energy within the resonance zone

$$d^3 = \frac{m}{e}\,\frac{\gamma^2}{16\pi e n_0 \omega\omega_0 \delta} \tag{17}$$

$$\Delta\Psi = -\int\limits_{z_p-d}^{z_p} E_\rho dz = \int\limits_0^1 4\pi e n_0 d^2 \exp\left(\frac{1}{2} - \alpha\xi^2\right)\frac{d\xi}{\sqrt{2\alpha}}$$

For comparison with experiment the dependence of the width of the resonance zone on the frequency, the nonuniformity gradient of the magnetic field δ, and the amplitude of the high-frequency field, related to the quantity γ, are of interest. Experiments should enable a judgment to be made on the correctness of the chosen model of plasma acceleration in the resonance zone, and on the part played by this zone in the overall energy increment of the plasma.

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